Long-distance effects on the $B \to X_s\gamma$ photon energy spectrum

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Abstract. We compute long-distance effects on the photon energy spectrum in inclusive radiative decays of B mesons using light-cone expansion and heavy-quark effective theory. We show that for sufficiently high photon energy, the leading nonperturbative QCD contribution is attributed to the distribution function. The distribution function is found to be universal in the sense that the same distribution function also encodes the leading nonperturbative contribution to inclusive semileptonic decays of B mesons at large momentum transfer. Some basic properties of the distribution function are deduced in QCD. Ways of extracting the distribution function directly from experiment and their implications are discussed. The theoretically clean methods for the determination of $|V_{ts}|$ are described.

1 Introduction

The study of the inclusive radiative decay $B \to X_s\gamma$, where X_s is any possible final state of total strangeness -1, is a broad subject with many areas of investigation. The decay is a flavor-changing neutral current process that is forbidden at tree level, only proceeding through what is called an electroweak penguin diagram [1–8] in the standard model. This is a one-loop graph inducing $b \to s$+photon with a W boson and a quark, predominantly the top quark, in the loop. Measurements of this rare process provide one of the most stringent experimental tests of the standard model at one-loop level. It can be used to determine the fundamental Cabibbo–Kobayashi–Maskawa (CKM) mixing matrix element $|V_{ts}|$ and test its unitarity, and may offer new insight into the nature of confinement.

Meanwhile, because the decay is highly suppressed in the standard model, it could be particularly sensitive to new physics beyond the standard model. Measurements of $B \to X_s\gamma$ decays would impose constraints on new physics models. In particular, new contributions from nonstandard particles replacing the standard model particles in the loop can be detected prior to a direct production of such new particles at much higher energies. A clear deviation from standard model expectations would signal new physics from supersymmetry, charged Higgs scalars, anomalous $WW\gamma$ couplings, etc.

Observation of the inclusive radiative decay $B \to X_s\gamma$ was reported in 1995 by the CLEO Collaboration, who obtained the first measurement of the branching ratio of $(2.32 \pm 0.57 \pm 0.35) \times 10^{-4}$ [9]. The ALEPH Collaboration has reported a measurement of the corresponding branching ratio of beauty hadrons produced at the Z resonance, obtaining the branching ratio of $(3.11 \pm 0.80 \pm 0.72) \times 10^{-4}$ [10]. Recently, the CLEO Collaboration has improved their measurement, using 60% additional data

and improved analysis techniques, and yielding a new preliminary result of the branching ratio of $(3.15 \pm 0.35 \pm 0.32 \pm 0.26) \times 10^{-4}$ [11]. Copious data samples from the rare decays $B \to X_s\gamma$ will soon be collected at B factories. The accuracy of the $B \to X_s\gamma$ measurement will be significantly improved shortly.

Significant progress has been made in the theoretical prediction of the branching ratio for $B \to X_s\gamma$ in the standard model. The next-to-leading order (NLO) calculation of the perturbative QCD corrections has been completed recently, combining the calculations of the matching conditions of the Wilson coefficients [12–15], the matrix elements [16–18], and the anomalous dimensions [19]. This achievement leads to a substantial reduction of the large renormalization scale dependence of the leading-order result [20–28]. The leading power corrections in $1/m_b^2$ [29, 30] and $1/m_c^2$ [31–35] have been calculated in the context of the heavy-quark expansion. In addition, the leading electroweak radiative corrections have recently been investigated [36, 37].

It is, however, not sufficient to have a reliable calculation of the total decay rate. Experimentally, only events in the high-energy region of the photon energy spectrum in $B \to X_s\gamma$ decays have been used to measure the branching ratio, so that backgrounds from other B-decay processes would be suppressed. An accurate and reliable theoretical description of the photon energy spectrum is essential in order to perform a fit to the experimental data and extrapolate to the total decay rate. The study of the photon energy spectrum is also quite interesting in its own right, since it can probe the decay dynamics in more detail than the total rate. Strong interaction effects can significantly modify the photon energy spectrum. The following fact well illustrates the situation. It is well known that the quark-level and virtual-gluon exchange processes $b \to s\gamma$ generate a trivial photon energy spectrum – a dis-

crete line at $E_\gamma = m_\mathrm{b}/2$ in the b-quark rest frame. It is two distinct effects, gluon-bremsstrahlung and hadronic bound-state effects, that spread out the spectrum. Gluon bremsstrahlung results in a long tail in the photon spectrum below $m_\mathrm{b}/2$. Bound-state effects lead to the extension of phase space from the parton level to the hadron level, stretch the spectrum downward below $m_\mathrm{b}/2$ and are solely responsible for populating the spectrum upward in the gap between the parton level endpoint $E_\gamma = m_\mathrm{b}/2$ and the hadron-level endpoint $E_\gamma = M_\mathrm{B}/2$.

The perturbative QCD corrections to the photon energy spectrum are known at order α_s [18], and the order $\alpha_\mathrm{s}^2\beta_0$ contribution has recently been computed [38] away from the endpoint $E_\gamma = m_\mathrm{b}/2$ in the perturbation expansion of the matrix elements. The relevant Wilson coefficients in the effective weak Hamiltonian are known to next-to-leading logarithmic accuracy [19] in renormalization-group improved perturbative QCD. Currently nonperturbative QCD effects comprise the potentially most serious source of theoretical error in the photon energy spectrum. Long-distance QCD effects are relevant for the improvement of the accuracy of an analysis and the understanding of theoretical uncertainties in the standard model, as well as for the search for new physics by the confrontation of standard model predictions with the data. In view of this, it is important to address the issue as rigorously and completely as possible. The Fermi motion of the b quark in the B meson has been taken into account in [16] by use of the phenomenological model by Altarelli, et al. [39]. A more fundamental treatment of bound-state effects based on the heavy-quark expansion has been developed by the resummation of an infinite set of leading-twist operators into a shape function [40–42]. Recently, an analysis of long-distance effects on the photon spectrum has been performed [37] along these lines, including for the first time the full NLO perturbative QCD corrections.

In this paper, we will compute the long-distance QCD contributions to the photon energy spectrum using light-cone expansion and heavy-quark effective theory (HQET). These ideas and techniques have originally been exploited in the theoretical description of inclusive semileptonic decays of beauty hadrons [43]. This approach has been related [44] to the aforementioned similar approach based on the resummation of the heavy-quark expansion [40–42]. In this paper, we extend our previous analyses to inclusive radiative decays of B mesons. We strive to present our approach in a more detailed and complete form.

Beauty hadrons are the heaviest hadrons actually formed, because the top quark is too heavy to build hadrons; this confers a special role to beauty hadrons. The beauty-hadron decays involve two large scales: the heavy beauty-hadron mass at the hadron level and the heavy b-quark mass at the parton level, both of which are much greater than the energy scale Λ_QCD which characterizes the strong interactions. The two large scales give rise to two techniques for dealing with nonperturbative QCD. Because of the heaviness of the decaying hadron, the decay dynamics may be dominated by the space-time separations in the neighborhood of the light cone. The light-cone expansion provides a formal and powerful way of organizing the nonperturbative QCD effects and singling out the leading term. On the other hand, the heavy-quark mass provides a large limit to construct an effective theory describing the heavy quark interacting with the gluons in the heavy hadron. This so-called heavy-quark effective theory has new (approximate) symmetries that were not manifested in the full QCD Lagrangian, and sets another framework for organizing and parametrizing nonperturbative effects that relates various phenomena (e.g., the hadron spectroscopy and weak decays of hadrons containing a single heavy quark) to a common set of parameters, so that it has great predictive power. It is certainly useful to calculate long distance QCD effects on the photon energy spectrum in B → $X_\mathrm{s}\gamma$ decays using light-cone expansion and heavy-quark effective theory.

The paper is organized as follows. In Sect. 2, we generalize the results of [43] and construct the light-cone expansion for the photon energy spectrum in B → $X_\mathrm{s}\gamma$ decays. We show that for sufficiently high photon energy, the leading nonperturbative QCD contribution is attributed to a distribution function. We define a new, gauge-invariant distribution function. The connection between the nonperturbative QCD effects in radiative and semileptonic inclusive B decays is subsequently established in Sect. 3. For the sake of completeness, we study the properties of the new distribution function in Sect. 4. We discuss the direct extraction of the distribution function from experiment and precise determinations of the CKM matrix elements in Sect. 5. We hope to motivate the measurement of the distribution function by pointing out its importance in inclusive B decays and the ways to measure it. Section 6 briefly summarizes the main results.

2 Light-cone expansion for B → $X_\mathrm{s}\gamma$ decays

We shall study inclusive radiative decays $\mathrm{B}(P) \to X_\mathrm{s}(p_X)$ $\gamma(p_\gamma)$. A suitable framework for doing that is an effective low-energy theory, obtained by integrating out the heavy particles, which in the standard model are the top quark and the W boson [23]. Since we shall concentrate on long-distance effects, it suffices for us to work at lowest order in electroweak interactions ignoring for the moment perturbative QCD and electroweak radiative corrections. To lowest order, B → $X_\mathrm{s}\gamma$ decays are governed by the effective weak Hamiltonian involving a magnetic dipole operator:

$$\mathcal{H}_\mathrm{eff} = \kappa \bar{s}\sigma^{\mu\nu} R\, b F_{\mu\nu}, \qquad (1)$$

where the coupling constant

$$\kappa = -\frac{G_\mathrm{F} e m_\mathrm{b}}{4\sqrt{2}\pi^2} V_\mathrm{tb} V_\mathrm{ts}^* C_7^{(0)}(M_\mathrm{W}) \qquad (2)$$

gauges the strength of the b→ $s\gamma$ transition, $R = (1 + \gamma_5)/2$ is the right-handed projection operator, and $F_{\mu\nu}$ is the electromagnetic field strength tensor. In (2), V_tb and V_ts are the CKM matrix elements, and the function $C_7^{(0)}(M_\mathrm{W})$ depends on the masses of the internal top quark and W boson, and takes the form [5]

$$C_7^{(0)}(M_W) = \frac{3x_t^3 - 2x_t^2}{4(x_t-1)^4}\ln x_t - \frac{8x_t^3 + 5x_t^2 - 7x_t}{24(x_t-1)^3} \qquad (3)$$

with $x_t = m_t^2/M_W^2$. We use m_b to denote the b-quark mass and M_B to denote the B-meson mass. It can be shown that contributions of the strange quark mass to the decay rate are suppressed by the very small factor m_s^2/m_b^2. Hence we neglect the strange quark mass unless noted otherwise.

The decay rate for B → $X_s\gamma$ is given by

$$d\Gamma = \frac{1}{2E_B}\frac{d^3\mathbf{p}_\gamma}{(2\pi)^3 2E_\gamma}\sum_{X_s,\epsilon}(2\pi)^4\delta^4(P - p_\gamma - p_X)$$
$$\times |\langle X_s(p_X)\gamma(p_\gamma,\epsilon)|\mathcal{H}_{\text{eff}}|B(P)\rangle|^2, \qquad (4)$$

where we sum over all possible final states of total strangeness -1 as well as the two transverse polarizations of the photon. We adopt the standard covariant normalization $\langle B|B\rangle = 2E_B(2\pi)^3\delta^3(\mathbf{0})$ for the B-meson state. The decay rate can be expressed in terms of a current commutator taken between the B-meson states which incorporates all nonperturbative QCD physics of the weak radiative B-meson decay. We find

$$d\Gamma = \frac{|\kappa|^2}{2E_B}\frac{d^3\mathbf{p}_\gamma}{(2\pi)^3 2E_\gamma}$$
$$\times \int d^4y\, e^{ip_\gamma \cdot y}\langle B\,|[J_\mu^\dagger(y), J^\mu(0)]|\,B\rangle, \qquad (5)$$

with the generalized current $J_\mu(y) = \bar{b}(y)[\gamma_\mu, p\!\!\!/_\gamma]Ls(y)$, where $L = (1-\gamma_5)/2$ is the left-handed projection operator.

The commutator in (5) has to vanish for space-like separations $y^2 < 0$ due to causality. Moreover, the behavior of the integral in (5) is determined by the integrand in domains with less rapid oscillations, i.e., $|p_\gamma \cdot y| \sim 1$, which implies $y^2 \lesssim 1/E_\gamma^2$. Combining these results we find that the dominant contribution to the integral in (5) results from the space-time region $0 \leq y^2 \lesssim 1/E_\gamma^2$. This implies that for sufficiently high photon energy, $E_\gamma \gg \Lambda_{\text{QCD}}$, the space-time separations in the neighborhood of the light cone $y^2 = 0$ dominate the decay dynamics. It is then appropriate to construct a light-cone expansion to calculate the photon energy spectrum in the high-energy region.

The decay-produced strange quark propagating in a background gluon field obeys the anticommutation relation

$$\{s(x), \bar{s}(y)\} = (i\partial\!\!\!/_x + m_s)i\Delta_s(x-y)U(x,y), \qquad (6)$$

with the Wilson link

$$U(x,y) = \mathcal{P}\exp[ig_s\int_y^x dz^\mu A_\mu(z)] \qquad (7)$$

between the quark fields at y and x, where $\mathcal{P}$ denotes path ordering. Here the strange-quark mass $m_s \neq 0$ is retained. The background gluon field A^μ is that generated by the remnants of the decaying B meson. $\Delta_q(y)$ is the Pauli–Jordan function of the form [45]

$$\Delta_q(y) = -\frac{i}{(2\pi)^3}\int d^4k\, e^{-ik\cdot y}\varepsilon(k^0)\delta(k^2 - m_q^2) \qquad (8)$$

$$= -\frac{1}{2\pi}\varepsilon(y^0)\delta(y^2) + \frac{m_q}{4\pi\sqrt{y^2}}\varepsilon(y^0)\theta(y^2)J_1(m_q\sqrt{y^2}).$$

Here $J_1(z)$ is the Bessel function of order 1. From (8) we see that $\Delta_q(y)$ is singular on the light cone and would also select out light-cone contributions were it not for high-frequency variations in the phase. Using the anticommutation relation (6), we find

$$[J_\mu^\dagger(y), J^\mu(0)]$$
$$= 16p_{\gamma\alpha}p_{\gamma\beta}[i\partial^\alpha i\Delta_s(y)]\bar{b}(0)\gamma^\beta RU(0,y)b(y). \qquad (9)$$

Substituting (9) in (5), it follows that

$$d\Gamma = \frac{|\kappa|^2}{2E_B}\frac{d^3\mathbf{p}_\gamma}{(2\pi)^3 2E_\gamma}8p_{\gamma\alpha}p_{\gamma\beta}\int d^4y\, e^{ip_\gamma\cdot y}[i\partial^\alpha i\Delta_s(y)]$$
$$\times \langle B|\bar{b}(0)\gamma^\beta U(0,y)b(y)|B\rangle. \qquad (10)$$

Note that the matrix element of $\bar{b}(0)\gamma^\beta\gamma_5 U(0,y)b(y)$ between the B-meson states vanishes by parity invariance in the strong interactions.

Now let us consider the matrix element of the bilocal operator in (10). The general tensor decomposition of it reads

$$\langle B|\bar{b}(0)\gamma^\beta U(0,y)b(y)|B\rangle$$
$$= 2[P^\beta F(y^2, y\cdot P) + y^\beta G(y^2, y\cdot P)], \qquad (11)$$

where $F(y^2, y\cdot P)$ and $G(y^2, y\cdot P)$ are in general functions of the two independent Lorentz scalars, y^2 and $y\cdot P$. Contracting both sides of (11) with y_β gives

$$\langle B|\bar{b}(0)y\!\!\!/U(0,y)b(y)|B\rangle$$
$$= 2[y\cdot PF(y^2, y\cdot P) + y^2 G(y^2, y\cdot P)]. \qquad (12)$$

In the domain of interest where $|p_\gamma\cdot y| \sim 1$ and $0 \leq y^2 \lesssim 1/E_\gamma^2$, we infer from (12) that the contribution of $G(y^2, y\cdot P)$ is suppressed by a factor $1/E_\gamma^2$ for the high photon energy considered here, as compared to that of $F(y^2, y\cdot P)$. In addition, we can make a light-cone expansion for $F(y^2, y\cdot P)$, which is justified for high photon energy as discussed above,

$$F(y^2, y\cdot P) = \sum_{n=0}^{\infty}\frac{(y^2)^n}{n!}\left[\frac{d^n F(y^2, y\cdot P)}{dy^{2n}}\right]_{y^2=0}. \qquad (13)$$

The nth term in the light-cone expansion is suppressed by $1/E_\gamma^{2n}$. Therefore, in the leading-twist approximation we then have

$$\langle B|\bar{b}(0)\gamma^\beta U(0,y)b(y)|B\rangle = 2P^\beta F(y^2 = 0, y\cdot P). \qquad (14)$$

The contributions from $F(y^2 \neq 0, y\cdot P)$ and $G(y^2, y\cdot P)$ represent higher twist effects. They become important only at the subleading level, an issue that will not be dwelt upon here.

On the light cone, $F(y^2 = 0, y\cdot P)$ can be projected out of the general decomposition (12):

$$F(y^2 = 0, y\cdot P) = \frac{1}{2y\cdot P}\langle B|\bar{b}(0)y\!\!\!/U(0,y)b(y)|B\rangle|_{y^2=0}. \quad (15)$$

The Fourier transform of $F(y^2 = 0, y \cdot P)$ defines the distribution function

$$f(\xi) = \frac{1}{2\pi} \int d(y \cdot P) \, e^{i\xi y \cdot P} F(y^2 = 0, y \cdot P) \qquad (16)$$

$$= \frac{1}{4\pi} \int \frac{d(y \cdot P)}{y \cdot P} \, e^{i\xi y \cdot P} \langle B|\bar{b}(0) \slashed{y} U(0, y) b(y)|B\rangle|_{y^2 = 0}.$$

The Wilson link is associated with the background gluon field, which ensures gauge invariance of the distribution function. The leading nonperturbative QCD contribution contained in $F(y^2 = 0, y \cdot P)$ is equivalently translated into the distribution function.

Using the inverse Fourier transform

$$F(y^2 = 0, y \cdot P) = \int d\xi \, e^{-i\xi y \cdot P} f(\xi), \qquad (17)$$

substituting (8) and (14) in (10), and then carrying out the phase space integration, we arrive at the photon energy spectrum in the B rest frame

$$\frac{d\Gamma(B \to X_s \gamma)}{dE_\gamma}$$

$$= \frac{G_F^2 \alpha m_b^2}{2\pi^4 M_B} |V_{tb} V_{ts}^*|^2 |C_7^{(0)}(M_W)|^2 E_\gamma^3 f\left(\frac{2E_\gamma}{M_B}\right). \quad (18)$$

Equation (18) holds in the leading-twist approximation, which is expected to be a good approximation for the energy of the emitted photon of around 1 GeV and above. The precise shape of the spectrum near the lower endpoint $E_\gamma = 0$ is not available to us, where the light-cone expansion is not applicable and higher twist terms give contributions of the same order as the leading-twist term. However, given the experimental [9–11] and theoretical [16,42,37] indications that the spectrum for $E_\gamma < 1$ GeV appears to be vanishingly small, a more accurate account of the rather small overall nonperturbative contributions in this region seems numerically unimportant. This is not unexpected, since the photon energy spectrum stemming from the quark-level and virtual-gluon exchange processes would only concentrate at $E_\gamma = m_b/2 \approx 2.45$ GeV, and gluon-bremsstrahlung and hadronic bound-state effects smear the spectrum about this point, but most of the decay rate remains at large values of E_γ. For practical purposes, this approach would nonetheless provide a realistic description of the photon spectrum over the full kinematic range $0 \le E_\gamma \le M_B/2 = 2.64$ GeV. Of course, one should keep in mind that for the photon spectrum near the lower endpoint, one has nothing from the approach to be verified or falsified.

Finally, let us discuss the relationship between our approach and the approach advocated in [40–42] based on the resummation of the heavy-quark expansion. By assuming $E_\gamma = m_b/2$ for the factor E_γ^3 in (18) we can reproduce the formulas for the $B \to X_s \gamma$ photon energy spectrum obtained in [41,42]. In that case, the photon energy is fixed to the value in the free-quark limit, instead of varying in its entire kinematic range from 0 to $M_B/2$.

3 Universality

In this section we compare the inclusive radiative decay with the inclusive semileptonic decays of B mesons $B \to X_f \ell \nu$ ($f = u, c$) in order to decipher the universal, process-independent structure in the relevant realm of hadron physics.

Inclusive semileptonic decays of B mesons at large momentum transfer carried by the W boson are also governed by the light-cone dynamics [43]. The nonperturbative QCD effects on the decays reside in the hadronic tensor

$$W_{\mu\nu} = -\frac{1}{2\pi} \int d^4y \, e^{iq \cdot y} \langle B|[j_\mu(y), j_\nu^\dagger(0)]|B\rangle, \qquad (19)$$

where the charged weak current $j_\mu(y) = \bar{f}(y)\gamma_\mu(1-\gamma_5)b(y)$. Calculating the charged weak current commutator in the same way as in the last section yields

$$\langle B| [j_\mu(y), j_\nu^\dagger(0)] |B\rangle = 2(S_{\mu\alpha\nu\beta} - i\varepsilon_{\mu\alpha\nu\beta}) [\partial^\alpha \Delta_f(y)]$$
$$\times \langle B|\bar{b}(0)\gamma^\beta U(0, y)b(y)|B\rangle, \quad (20)$$

where $S_{\mu\alpha\nu\beta} = g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha} - g_{\mu\nu}g_{\alpha\beta}$. We observe here the very same matrix element, $\langle B|\bar{b}(0)\gamma^\beta U(0, y)b(y)|B\rangle$, entering the decay rate for the inclusive semileptonic decays of B mesons.

The twist expansion of the bilocal operator matrix element then leads to the expression of the differential decay rate in the B rest frame in terms of the distribution function $f(\xi)$ defined in (16):

$$\frac{d^3\Gamma(B \to X_f \ell \nu)}{dE_\ell dq^2 dq^0}$$
$$= \frac{G_F^2 |V_{fb}|^2}{4\pi^3 M_B} \frac{q^0 - E_\ell}{\sqrt{|\mathbf{q}|^2 + m_f^2}}$$
$$\times \left\{ f(\xi_+)(2\xi_+ E_\ell M_B - q^2) - (\xi_+ \to \xi_-) \right\}, \quad (21)$$

where

$$\xi_\pm = \frac{q^0 \pm \sqrt{|\mathbf{q}|^2 + m_f^2}}{M_B}, \qquad (22)$$

and m_f is the mass of the decay-produced quark, which is the up (charm) quark with $f = u(c)$. All the lepton masses have been neglected. Therefore, the distribution function describes leading long-distance QCD effects not only in inclusive radiative B decays, but also in inclusive semileptonic B decays, and is thus a universal function. This universality originates from the fact that the primary object of analysis in long-distance effects is the same bilocal operator matrix element dictated by the light-cone dynamics.

The b-quark distribution function $f(\xi)$ defined in (16) differs from the definition given in [43]. The difference is that the distribution function defined here is gauge-invariant, and its inverse Fourier transform is exactly equal to the leading-twist term $F(y^2 = 0, y \cdot P)$, whereas the distribution function of [43] is defined in the light-cone

gauge and its inverse Fourier transform is equal to $F(y^2 = 0, y \cdot P)$ in the leading-twist approximation. However, we note that the expressions for the inclusive semileptonic B-decay rates derived previously [43] in terms of the distribution function remain formally the same if the distribution function defined in this paper is used. This is exemplified by (21) for the triple differential decay rate.

4 Properties of the distribution function

The b-quark distribution function is an important physical quantity which summarizes leading long-distance effects on inclusive B-decay processes. Equation (18) shows that the photon energy spectrum depends strongly on the distribution function. In Sect. 2 we defined the b-quark distribution function in QCD by modifying the definition given in [43]. In this section, we rederive several important properties of the distribution function. Since it is gauge-invariant, it can be calculated in any gauge. For simplicity, we shall choose the light-cone gauge in the following discussion so that the Wilson link becomes the identity operator.

It is helpful to introduce the null vector $n^\mu = (1, 0, 0, -1)$ at this step, such that the light-like vector $y^\mu = tn^\mu$, where t is a parameter. Using this notation, the distribution function defined in (16) can be rewritten as follows:

$$f(\xi) = \frac{1}{2\pi} \int \mathrm{d}t\, \mathrm{e}^{\mathrm{i}\xi tn \cdot P} \langle B|b_+^\dagger(0)b_+(tn)|B\rangle, \quad (23)$$

where $b_+ = P_+ b$ is the "good" component projected out of the b-quark field, with the projection operator $P_+ = (1 + \gamma^0\gamma^3)/2$. Inserting a complete set of hadronic states between quark fields and translating the tn dependence out of the field, we find

$$f(\xi) = \sum_m \delta[n \cdot (P - \xi P - p_m)]|\langle m|b_+(0)|B\rangle|^2. \quad (24)$$

So the distribution function obeys positivity. The state $|m\rangle$ is physical and must have $0 \le p_m^0 \le E_B$, thus $f(\xi) = 0$ for $\xi \le 0$ or $\xi \ge 1$. Therefore, the support of the distribution function reads $0 \le \xi \le 1$. Moreover, one can observe from (24) that $f(\xi)$ is the probability of finding a b quark with momentum ξP inside the B meson. This is the familiar probabilistic interpretation of the parton model [46] for inclusive B decays.

From (17) and (11) it is straightforward to show that

$$\int_0^1 \mathrm{d}\xi\, f(\xi) = F(y = 0) = 1, \quad (25)$$

because b-quantum number conservation implies

$$\langle B|\bar{b}\gamma^\mu b|B\rangle = 2P^\mu. \quad (26)$$

Thus the distribution function is exactly normalized to unity. The normalization does not get renormalized as a consequence of b-quantum number conservation.

To understand B-meson bound-state effects, it is instructive to consider first of all the form of the b-quark distribution function in the free-quark limit. In this limit, the B meson and the b quark in it move together with the same velocity: $p_b/m_b = P/M_B \equiv v$ and the free Dirac field $b(y) = \mathrm{e}^{-\mathrm{i}y \cdot p_b}b(0)$, so from (16) it follows that

$$f_{\mathrm{free}}(\xi) = \delta(\xi - m_b/M_B). \quad (27)$$

Substituting (27) in (18), it consistently reduces to the free quark decay spectrum in the rest frame of the b quark

$$\frac{\mathrm{d}\Gamma(b \to s\gamma)}{\mathrm{d}E_\gamma} = \frac{G_F^2 \alpha m_b^5}{32\pi^4}|V_{tb}V_{ts}^*|^2|C_7^{(0)}(M_W)|^2 \delta\left(E_\gamma - \frac{m_b}{2}\right), \quad (28)$$

which is a discrete line at $E_\gamma = m_b/2$. Conversely, the physical photon spectrum is obtained from a convolution of the hard perturbative spectrum with the soft nonperturbative distribution function $f(\xi)$:

$$\frac{\mathrm{d}\Gamma(B \to X_s\gamma)}{\mathrm{d}E_\gamma} = \int_0^1 \mathrm{d}\xi\, f(\xi)\frac{\mathrm{d}\Gamma(b \to s\gamma, p_b = \xi P)}{\mathrm{d}E_\gamma}, \quad (29)$$

where the b-quark momentum p_b in the perturbative spectrum is replaced by ξP. The bound-state smearing of the photon energy spectrum is then reflected in the deviation of the true distribution function from the delta function form. The quantitative analysis of this deviation is the subject of the rest of this section.

Additional properties of the distribution function can be deduced by means of operator product expansion and heavy-quark effective theory. The derivation proceeds in essentially the same manner as in [43], but for the different definition of the distribution function. We present the corresponding derivation for the new distribution function below for completeness.

Since the b quark inside the B meson behaves as almost free due to its large mass, relative to which its binding to the light constituents is weak, one can extract the large space-time dependence

$$b(y) = \mathrm{e}^{-\mathrm{i}m_b v \cdot y}b_v(y). \quad (30)$$

A Taylor expansion of the field in a gauge-covariant form relates the bilocal and local operators. This leads to an operator product expansion

$$\bar{b}(0)\gamma^\beta b(y) = \mathrm{e}^{-\mathrm{i}m_b v \cdot y} \sum_{n=0}^\infty \frac{(-\mathrm{i})^n}{n!} y_{\mu_1} \cdots y_{\mu_n}$$
$$\times \bar{b}_v(0)\gamma^\beta k^{\{\mu_1} \cdots k^{\mu_n\}}b_v(0), \quad (31)$$

where $k_\mu = \mathrm{i}D_\mu = \mathrm{i}(\partial_\mu - \mathrm{i}g_s A_\mu)$ and the symbol $\{\cdots\}$ means symmetrization with respect to the enclosed indices. Using Lorentz covariance, one can express the matrix element of the local operator on the right-hand side of (31) between the B-meson states in terms of the B-meson momentum:

$$\langle B|\bar{b}_v(0)\gamma^\beta k^{\{\mu_1} \cdots k^{\mu_n\}}b_v(0)|B\rangle$$

$$= 2\left(C_{n0}P^\beta P^{\mu_1}\cdots P^{\mu_n} + \sum_{i=1}^n M_B^2 C_{ni}g^{\beta\mu_i}P^{\mu_1}\right.$$

$$\left.\cdots P^{\mu_{i-1}}P^{\mu_{i+1}}\cdots P^{\mu_n}\right) + \text{terms with } g^{\mu_i\mu_j}. \quad (32)$$

The terms with $g^{\mu_i\mu_j}$ drop out on the light cone. Substituting (31) and (32) into (16) yields

$$f(\xi) = \sum_{n=0}^\infty \frac{(-1)^n}{n!}C_{n0}\delta^{(n)}\left(\xi - \frac{m_b}{M_B}\right). \quad (33)$$

Therefore we obtain the moment relation for the distribution function

$$M_n(m_b/M_B) = C_{n0}, \quad (34)$$

where the nth moment about a point $\tilde\xi$ of the distribution function is in general defined by

$$M_n(\tilde\xi) = \int_0^1 d\xi(\xi - \tilde\xi)^n f(\xi). \quad (35)$$

By definition, $M_0(\tilde\xi) = C_{00} = 1$. The moment relation (34) relates the moments of the distribution function to the matrix elements of the local operators in (32).

We invoke heavy-quark effective theory to evaluate further expansion coefficients in (32). In this effective theory, the QCD b-quark field $b(y)$ is related to its HQET counterpart $h(y)$ by means of an expansion in powers of $1/m_b$:

$$b(y) = e^{-im_b v\cdot y}\left[1 + \frac{i\slashed{D}}{2m_b} + O(\frac{\Lambda_{QCD}^2}{m_b^2})\right]h(y). \quad (36)$$

The effective Lagrangian takes the form

$$\mathcal{L}_{HQET} = \bar h iv\cdot Dh + \bar h\frac{(iD)^2}{2m_b}h + \bar h\frac{g_s G_{\alpha\beta}\sigma^{\alpha\beta}}{4m_b}h + O(\frac{1}{m_b^2}), \quad (37)$$

where $g_s G^{\alpha\beta} = i[D^\alpha, D^\beta]$ is the gluon field-strength tensor. Only the first term in (37) remains in the $m_b \to \infty$ limit, which respects the heavy-quark spin and flavor symmetries. The other two terms give the $1/m_b$ corrections: The second term violates the heavy-flavor symmetry, while the third term violates both the spin and flavor symmetries. Using the method described in [47–49] to relate matrix elements of local operators in full QCD to those in the HQET, the expansion coefficients C_{nl} in (32) can be expressed in terms of the HQET parameters. The nonperturbative QCD effects can, in principle, be calculated in a systematic manner. In this formalism, the moment $M_n(m_b/M_B)$ is expected to be of order $(\Lambda_{QCD}/m_b)^n$. A few coefficients have been evaluated and the results are [43]

$$C_{10} = \frac{5m_b}{3M_B}E_b + O(\Lambda_{QCD}^3/m_b^3), \quad (38)$$

$$C_{11} = -\frac{2m_b}{3M_B}E_b + O(\Lambda_{QCD}^3/m_b^3), \quad (39)$$

$$C_{20} = \frac{2m_b^2}{3M_B^2}K_b + O(\Lambda_{QCD}^3/m_b^3), \quad (40)$$

$$C_{21} = C_{22} = 0, \quad (41)$$

where $E_b = K_b + G_b$ and K_b and G_b are the dimensionless HQET parameters of order $(\Lambda_{QCD}/m_b)^2$, which are often referred to by the alternate names $\lambda_1 = -2m_b^2 K_b$ and $\lambda_2 = -2m_b^2 G_b/3$, defined as

$$\lambda_1 = \frac{1}{2M_B}\langle B|\bar h(iD)^2 h|B\rangle, \quad (42)$$

$$\lambda_2 = \frac{1}{12M_B}\langle B|\bar h g_s G_{\alpha\beta}\sigma^{\alpha\beta}h|B\rangle. \quad (43)$$

The parameter λ_2 can be extracted from the $B^* - B$ mass splitting: $\lambda_2 = (M_{B^*}^2 - M_B^2)/4 = 0.12$ GeV^2. The parameter λ_1 suffers from large uncertainty.

Thus two sum rules for the distribution function can be derived according to the moment relation (34). They determine up to order $(\Lambda_{QCD}/m_b)^2$ the mean value μ and the variance σ^2 of the distribution function, which characterize the location of the "center of mass" of the distribution function and the square of its width, respectively:

$$\mu = \frac{m_b}{M_B}\left(1 + \frac{5E_b}{3}\right), \quad (44)$$

$$\sigma^2 = \left(\frac{m_b}{M_B}\right)^2\left[\frac{2K_b}{3} - \left(\frac{5E_b}{3}\right)^2\right], \quad (45)$$

with the definitions

$$\mu \equiv M_1(0) = \tilde\xi + M_1(\tilde\xi), \quad (46)$$
$$\sigma^2 \equiv M_2(\mu) = M_2(\tilde\xi) - M_1^2(\tilde\xi). \quad (47)$$

The mean value and variance specify the primary shape of the distribution function. Therefore, our evaluation comes to the conclusion that the distribution function $f(\xi)$ is sharply peaked around $\xi = \mu \approx m_b/M_B$, close to 1, and its width of order Λ_{QCD}/M_B is narrow.

The sum rules given in (44) and (45) quantify the deviation of the true distribution function from the delta function form due to bound-state effects. Choosing the parameters $m_b = 4.9$ GeV and $\lambda_1 = -0.5$ GeV^2 for the purpose of orientation, we obtain from (44) and (45) $\mu = 0.933$ and $\sigma^2 = 0.006$. By contrast, the mean value $\mu = m_b/M_B = 0.928$ and the variance $\sigma^2 = 0$ in the free-quark limit.

The results for the mean value and the variance given in (44) and (45) are at variance with those of [43] because of the different definitions of the distribution function. However, the corresponding numerical shifts are found to be so small that the phenomenological impacts of these differences are negligible.

Nonperturbative QCD methods such as lattice simulation and QCD sum rules could help determine further the form of the distribution function. The distribution function could also be extracted directly from experiment, as we shall discuss below.

5 Measurements of the distribution function and determinations of the CKM matrix elements

The universality of the distribution function discussed in Sect. 3 enhances the predictive power of the approach: The distribution function can be extracted from measurements of one process and then used to make predictions in all other processes in a model-independent manner. In this section, we start by exploring how to extract the distribution function from experiment. Then we will investigate the theoretically clean methods for the determination of the CKM matrix element $|V_{ts}|$.

The spectrum result (18) can be cast in the following form:

$$|V_{tb}V_{ts}^*|^2 f\left(\frac{2E_\gamma}{M_B}\right) = \frac{2\pi^4 M_B}{G_F^2 \alpha m_b^2 |C_7^{(0)}(M_W)|^2} \frac{1}{E_\gamma^3}$$
$$\times \frac{d\Gamma(B \to X_s\gamma)}{dE_\gamma}. \tag{48}$$

This immediately implies that the distribution function can be extracted directly from a measurement of the photon energy spectrum upon implementation of perturbative QCD and electroweak radiative corrections, which have so far been ignored in this paper. The experimental data on the photon energy spectrum are already available [9–11], but too limited by statistics for a meaningful extraction of the distribution function to be made. Forthcoming very large data samples from high-luminosity B factories promise a direct extraction of the distribution function with reasonable accuracy. Such a determination of the distribution function would help substantially to reduce the theoretical uncertainties in the descriptions of both semileptonic and radiative inclusive decays of B mesons. The distribution function extracted from B → $X_s\gamma$ can be applied, for example, in the calculations of the lepton energy spectrum and the hadronic invariant mass spectrum in charmless inclusive semileptonic decays B → $X_u\ell\nu$, so that the precision of the $|V_{ub}|$ determination from these spectra can be improved.

The distribution function can also be extracted directly from the measurements of the differential decay rates as a function of the scaling variable ξ_+ [note that ξ_+ is a different kinematic variable for B → $X_u\ell\nu$ and B → $X_c\ell\nu$, defined in (22)] in the inclusive semileptonic decays of B mesons [50]:

$$|V_{fb}|^2 f(\xi_+) = \frac{192\pi^3}{G_F^2 M_B^5} \frac{1}{\xi_+^5 \Phi(r_f/\xi_+)} \frac{d\Gamma(B \to X_f\ell\nu)}{d\xi_+}, \tag{49}$$

where $r_f = m_f/M_B$ (f = u, c) and $\Phi(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4\ln x$. Such a determination of the distribution function would also benefit the theoretical descriptions of both semileptonic and radiative inclusive decays of B mesons. In particular, once such an extraction of the distribution function is available, it can in turn be applied as an independent input in the analysis of the

B → $X_s\gamma$ photon energy spectrum. This would substantially improve the standard model predictions and increase the sensitivity to new physics. Therefore, this extraction of the distribution function is valuable in the search for additional decay mechanisms beyond the standard model in B → $X_s\gamma$ decays. The experimental technique of neutrino reconstruction could well make this way of extracting the distribution function experimentally feasible. If the neutrino can be reconstructed kinematically by the inferring of its four-momentum from the missing energy and missing momentum in each event, then it is possible to measure the scaling variable ξ_+.

It is important to note that the B → $X_s\gamma$ photon energy spectrum and the B → $X_f\ell\nu$ spectra $d\Gamma/d\xi_+$ share a common feature – namely, they offer the intrinsically most sensitive probe of long-distance strong interactions, because these spectra correspond to a discrete line solely on kinematic grounds in the absence of gluon-bremsstrahlung and long-distance strong interactions. Indeed, our calculation based on the light-cone expansion shows that they are explicitly proportional to the nonperturbative distribution function. Therefore, the shapes of these spectra directly reflect the inner long-distance dynamics of the reactions, and measurements of these spectra are ideally suited for direct extraction of the distribution function from experiment. This salient feature also makes the b→ u scaling variable ξ_+ unique to give a very efficient discrimination between B → $X_u\ell\nu$ and B → $X_c\ell\nu$ events [50], even better than the hadronic invariant mass.

Given the difficulty in distinguishing t→ s decays from the dominant t→ b decay mode, it is hard to determine $|V_{ts}|$ from studies of top-quark decays. Based on the results obtained in this paper and in [50], we propose a new strategy for extracting $|V_{ts}|$ that is largely free of hadronic uncertainties. The idea is to use the known normalization of the distribution function or the cancellation of the distribution function in the ratio of the decay rates to eliminate the dominant hadronic uncertainties. The most straightforward and best way to eliminate the dependence on the distribution function is to resort again to the B → $X_s\gamma$ photon energy spectrum and the B → $X_{u,c}\ell\nu$ spectra $d\Gamma/d\xi_+$ since they are proportional to the distribution function.

Integrating (48) over E_γ and using the normalization condition (25) yields

$$|V_{tb}V_{ts}^*|^2 = \frac{4\pi^4}{G_F^2 \alpha m_b^2 |C_7^{(0)}(M_W)|^2}$$
$$\times \int_0^{M_B/2} dE_\gamma \frac{1}{E_\gamma^3} \frac{d\Gamma(B \to X_s\gamma)}{dE_\gamma}. \tag{50}$$

Thus the known normalization of the distribution function allows a model-independent determination of $|V_{tb}V_{ts}^*|$ from a measurement of the weighted integral of the photon energy spectrum. This determination can be taken as a measurement of $|V_{ts}|$ by the use of $|V_{tb}| = 0.9993$ from unitarity of the CKM matrix, which holds to a very high accuracy [51]. The advantage of this determination of $|V_{ts}|$ is that the dominant hadronic uncertainty has been

avoided, which may provide one of the most precise determinations of $|V_{\rm ts}|$.

As in the case of B → $X_s\gamma$ discussed above, one can get rid of the distribution function by integrating (49) over ξ_+ [50]:

$$|V_{f\rm b}|^2 = \frac{192\pi^3}{G_{\rm F}^2 M_{\rm B}^5} \int_{r_f}^1 {\rm d}\xi_+ \frac{1}{\xi_+^5 \Phi(r_f/\xi_+)} \frac{{\rm d}\Gamma({\rm B} \to X_f\ell\nu)}{{\rm d}\xi_+}. \tag{51}$$

Thus, the precise determinations of $|V_{\rm ub}|$ and $|V_{\rm cb}|$ can be obtained from the measurements of the weighted integrals of the differential decay rates as functions of ξ_+ in B → $X_u\ell\nu$ and B → $X_c\ell\nu$, respectively.

Alternatively, when we take the ratio of the differential decay rates, the distribution function cancels. From (48) and (49), we obtain

$$\left|\frac{V_{\rm tb}V_{\rm ts}^*}{V_{f\rm b}}\right|^2 = \frac{\pi M_{\rm B}}{3\alpha m_{\rm b}^2 |C_7^{(0)}(M_{\rm W})|^2} E_\gamma^2 \Phi(r_f/\xi_+)$$
$$\times \frac{{\rm d}\Gamma({\rm B} \to X_s\gamma)/{\rm d}E_\gamma}{{\rm d}\Gamma({\rm B} \to X_f\ell\nu)/{\rm d}\xi_+}\bigg|_{E_\gamma = M_{\rm B}\xi_+/2} , \tag{52}$$

which may be useful to provide a theoretically clean determination of $|V_{\rm ts}/V_{f\rm b}|$. By the same token, an analogous expression has been derived in [50] for B → $X_{u,c}\ell\nu$, which can be used to measure $|V_{\rm ub}/V_{\rm cb}|$ to a high precision. These determinations of the CKM matrix elements rely on the universality of the distribution function, in contrast to the method described above by virtue of the known normalization of the distribution function. The compatibility between the various measurements will test the universality of the distribution function in the inclusive semileptonic and radiative decays of B mesons within the standard model.

6 Summary

In this paper we have calculated the long-distance effects on the photon energy spectrum in B → $X_s\gamma$ decays. We have demonstrated on the basis of light-cone expansion that the leading nonperturbative QCD contribution in inclusive radiative B decays with emission of a sufficiently high energy photon resides in the distribution function. The distribution function is defined by Fourier transformation of the matrix element of the non-local b-quark operators separated along the light cone. We have found that the distribution function is universal in the sense that the same distribution function also summarizes the leading nonperturbative QCD contribution in inclusive semileptonic B decays at large momentum transfer.

Although long-distance strong interactions responsible for the distribution function preclude a complete calculation of it at present, we have deduced some of its basic properties in QCD. The distribution function is gauge-invariant and obeys positivity. It has a support between 0 and 1 and is exactly normalized to unity because of b-quantum number conservation. It contains the free quark

decay as a limiting case with $f_{\rm free}(\xi) = \delta(\xi - m_{\rm b}/M_{\rm B})$. The distribution function $f(\xi)$ can be interpreted as the probability of finding a b quark with momentum ξP inside the B meson with momentum P. In addition, we have evaluated the mean and variance of the distribution function using the techniques of operator product expansion and heavy-quark effective theory. They specify the primary shape of the distribution function and quantify the deviation from the delta function form in the free quark limit.

The b-quark distribution function for the B meson is a key object. Like the well-known parton distribution functions for the nucleon in deeply inelastic lepton–nucleon scattering, the knowledge of the b-quark distribution function would help us greatly in understanding the nature of confinement and the structure of the B meson. One should try to calculate it using nonperturbative QCD methods such as lattice simulation and QCD sum rules. On the other hand, the distribution function can be extracted directly from experimental data. The underlying common structure of long-distance strong interactions correlates the B → $X_s\gamma$ and B → $X_f\ell\nu$ processes. The universality of the distribution function implies that once it is measured from one process, it can be used to make predictions in all other processes in a model-independent manner. We have discussed the direct extraction of the distribution function from the measurements of the B → $X_s\gamma$ photon spectrum or the B → $X_f\ell\nu$ spectra ${\rm d}\Gamma/{\rm d}\xi_+$. We stress that these decay spectra are unique in that they offer the intrinsically most sensitive probe of long-distance strong interactions. The experimental extraction of the distribution function will lead to a significant improvement of the theoretical description of the B → $X_s\gamma$ photon energy spectrum, which is very important for seeking new physics. The extracted distribution function will also improve the theoretical description of inclusive semileptonic B decays, allowing for more precise determinations of $|V_{\rm ub}|$ and $|V_{\rm cb}|$. Moreover, a confrontation of experimental extraction of the distribution function with QCD predictions will be a test of our understanding of the B-meson structure and nonperturbative techniques. A direct extraction of the distribution function will therefore be an important aspect in future experiments in inclusive B-meson decays.

Measurements of B → $X_s\gamma$ decays can be used to determine the CKM matrix element $|V_{\rm ts}|$. We have described the theoretically clean methods for the determinations of the CKM matrix elements by use of the known normalization of the distribution function or the cancellation of the distribution function in the ratio of the differential decay rates, thereby avoiding the dominant hadronic uncertainties. The residual hadronic uncertainty in $|V_{\rm ts}|$ due to higher-twist, power-suppressed corrections is expected to be at the level of one percent. With hadronic uncertainties well under control, these methods eventually will yield the most accurate value of $|V_{\rm ts}|$, which is, on the other hand, probably one of the most reliable quantities to signal new physics in B → $X_s\gamma$. The agreement of $|V_{\rm ts}|$ extracted from the flavor-changing neutral current process B → $X_s\gamma$ with the value obtained from the direct mea-

surements plus unitarity under the assumption that the standard model is valid can be used to impose constraints on new physics models.

The universality of the distribution function in radiative and semileptonic inclusive decays of B mesons is valid only to the leading order in the light-cone expansion. To what extent this is a good approximation can be tested experimentally in a variety of ways in the B → $X_s\gamma$, B → $X_u\ell\nu$ and B → $X_c\ell\nu$ processes. Higher-twist effects may be numerically sizeable in some region of phase space, especially in the resonance domain. Their quantitative consequences deserve a thorough investigation.

The calculation of long-distance effects presented in this paper must be combined with the perturbative QCD and electroweak radiative corrections to give a detailed theoretical description of the photon energy spectrum. Considerable care must be exercised in including radiative corrections, which involves the interplay between perturbative and nonperturbative interactions. A detailed analysis of the issue will appear elsewhere. The theoretical development towards a treatment of inclusive radiative decays of B mesons from first principles, in conjunction with precision measurements made possible by B factories in coming years, will make a decisive test of the standard model at one-loop level and, more excitingly, may corroborate the existence of new physics in inclusive radiative decays of B mesons.

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